



## Notice for the PhD Viva Voce Examination

Ms Priyanka Pandey (Registration Number: 1740090), PhD scholar at the School of Sciences, CHRIST (Deemed to be University), Bangalore will defend her PhD thesis at the public viva-voce examination on Wednesday, 15 November, 2023 at 10.30 am in Room No. 044, Ground Floor, R & D Block, CHRIST (Deemed to be University), Bengaluru - 560029.

- Title of the Thesis** : **A Study on Graph Colouring with Distance Constraints**
- Discipline** : **Mathematics**
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- External Examiner (Within Karnataka)** : **Dr Dhananjayamurthy B V**  
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The members of the Research Advisory Committee of the Scholar, the faculty members of the Department and the School, interested experts and research scholars of all the branches of research are cordially invited to attend this open viva-voce examination.

**Registrar**

**Place:** Bengaluru

**Date:** 06 November 2023

## ABSTRACT

In this dissertation, we have studied the variations of graph colouring based on distance constraints. For a given set  $T$  of non-negative integers including zero and a positive integer  $k$ , the  $L(T, 1)$ -colouring of a graph  $G = (V, E)$  is a function  $c : V(G) \rightarrow \{0, 1, 2, \dots, k\}$  such that  $|c(u) - c(v)| \notin T$  if the distance between  $u$  and  $v$  is 1 and  $|c(u) - c(v)| \geq 1$  whenever  $u$  and  $v$  are at distance 2. The  $L(T, 1)$ -span,  $\lambda_{T,1}(G)$  is the smallest positive integer  $k$  such that  $G$  admits an  $L(T, 1)$ -Colouring. We have determined the  $L(T, 1)$ -span for some classes of graphs for set  $T$  whose elements are arranged in arithmetic progression. Further, for any general set  $T$ , we have found the bound for  $L(T, 1)$ -span of a few classes of graphs. We use Python programming to colour certain classes of graphs concerning  $L(T, 1)$ -colouring and find the value of  $L(T, 1)$ -span. Next, we have explored equitable fractional open neighbourhood colouring, which is an extension of a specific variation of  $L(h, k)$ -Colouring for  $h = 0$  and  $k = 1$ . For a positive integer  $p$ , equitable fractional open neighbourhood colouring of a graph  $G$  is an assignment of positive integers to the vertices of  $G$  such that for each vertex  $v \in V(G)$ , vertices of  $N(v)$  receives at least  $\lfloor \frac{|N(v)|}{p} \rfloor$  distinct colours and  $N(v)$  can be partitioned into  $k$ -classes  $V_1, V_2, \dots, V_k$  such that  $||V_i| - |V_j|| \leq 1$  for every  $i \neq j$  and  $1 \leq k \leq n$ . The minimum number of colours required to colour  $G$  such that it admits equitable fractional open neighbourhood colouring for a fixed  $p$  is called the equitable fractional open neighbourhood chromatic number,  $\chi_{eq \frac{onc}{p}}(G)$ . We have studied some properties of equitable fractional open neighbourhood colouring and explored some classes of graphs which admit equitable fractional open neighbourhood colouring with  $\lfloor \frac{\Delta(G)}{p} \rfloor$  colours. Further, we have introduced and examined a variation of perfect graphs,  $\chi_{onc}$ -perfect graphs, with respect to equitable fractional open neighbourhood colouring for the special case of  $p = 1$ . We have characterised graphs such that for every induced subgraph  $H$  of  $G$ ,  $\chi_{onc}(H) = \omega(H)$ , where  $\chi_{onc}(H)$  is open neighbourhood chromatic number of  $H$ . Moreover, we have found the classes of graphs  $G$  such that for every induced subgraph  $H$  of  $G$ ,  $\chi_{onc}(H) = \Delta(H)$ . In addition to this, we have derived a few more related results.

We have concluded the study by investigating the colouring which takes into account the average distance of the graphs. We have obtained the average distance colouring number  $\chi_\mu(G)$  of some graph classes with the help of derivations of the average distance of those classes of graphs. Finally, we have characterised graphs which require  $n$  distinct colours with respect to average distance colouring. We have also presented a greedy algorithm to colour any graph  $G$  with average distance constraint.

Keywords: Graph colouring,  $T$ -colouring,  $L(h, k)$ -colouring,  $L(T, 1)$ -colouring, colour span, equitable fractional open neighbourhood colouring,  $\chi_{onc}$ -perfect graphs, average distance, average distance colouring.

### Publications:

1. P. Pandey and M. Joseph, "Further results on  $L(T, 1)$ -Colouring of Graphs", Malaya Journal of Matematik vol. 5, no. 1, 121-125, 2020.
2. P. Pandey and M. Joseph, "On  $L(T, 1)$ -Colouring of certain classes of graphs" in IJSTR, vol. 9, no. 3, 6724-6731, 2020